Dephasing of coupled spin qubit system during gate operations due to background charge fluctuations

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It has been proposed that a quantum computer can be constructed based on electron spins in quantum dots or based on a superconducting nanocircuit. During two-qubit operations, the fluctuation of the coupling parameters is a critical factor. One source of such fluctuation is the stirring of the background charges. We focused on the influence of this fluctuation on a coupled spin qubit system. The induced fluctuation in exchange coupling changes the amount of entanglement, fidelity, and purity. In our previous study, the background charge fluctuations were found to be an important channel of dephasing for a single Josephson qubit.

Among the various proposals for quantum computation, quantum bits (qubits) in solid state materials, such as superconducting Josephson junctions[1] and quantum dots,[2–4] have the advantage of scalability. Proposals to implement a quantum computer using superconducting nanocircuits are proving to be very promising,[5–9] and several experiments have already highlighted the quantum properties of these devices.[10–12] Such a coherent-two-level system constitutes a qubit and the quantum computation can be carried out as the unitary operation functioning on the multiple qubit system. Essentially, this quantum coherence must be maintained during computation. However, dephasing is hard to avoid due to the system's interaction with the environment. The dephasing is characterized by the dephasing time T_2, and various environments can cause dephasing.

Background charge fluctuations (BCFs) have been observed in various kinds of systems.[13–16] In nanoscale systems, BCFs are electrostatic potential fluctuations arising due to the dynamics of an electron, or hole, on a charge trap. In particular, the charges at charge traps fluctuate with the Lorentzian spectrum form, which is called random telegraph noise in the time domain.[16, 17] The random distribution of the positions of such dynamical charge traps and their time constants leads to BCFs or $1/f$ noise.[18] In solid-state charge qubits, these BCFs result in a dynamical electrostatic disturbance and hence, dephasing. It should be noted that this dephasing process does not mean the qubit being entangled with the environment,

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but rather, that the stochastically evolution of an external classical field is suppressing the density matrix elements of the qubit after averaging out over statistically distributed samples.

We had shown that BCFs are important channel of dephasing for a single Josephson charge qubit system.[19, 20] In the present study, we investigate the effect of BCFs on the two-qubit-gate operation. To construct a controllable quantum computer, one requires the suppression of dephasing and accurate universal quantum gate which consists of single qubit operations and two-qubit operations. Therefore, to address these manipulations, we examine the dephasing of coupled qubit system, which is experimentally current topic and is urgent to analyze what cause of dephasing is important in these systems.

Recently, it has been shown that the interaction between electron spin in a quantum dot and environments is weak,[21, 22] and one can expect a very long dephasing time of an electron spin. Therefore, the proposal of quantum computer using electron spin in a quantum dot is promising.[4] For the electron spin qubit, however, the effects of ; the fluctuation of local magnetic field,[4, 23, 24] and the spin-orbit interaction[24] are important. Moreover, the fluctuation in the exchange coupling is important during the gate operation, because, for two-qubit gate operation, one uses the exchange interaction between the two quantum dots. This type of dephasing occurs when the charge of traps change the coulomb energy of two-qubit state and fluctuate the exchange energy between the two qubits. Therefore, we investigate the effect of BCFs on the two-qubit gate operation.

We examine the time evolution of two-qubit density matrix which obeys the two-qubit Hamiltonian $H = J(t)\mathbf{S_1} \cdot \mathbf{S_2}$, where $J(t)$ depends on time, and we neglect the magnetic field on each qubit system. We examine the amount of fluctuation in exchange coupling.(Fig1.)

We define the quantum variables of two qubits' coordinate as $\vec{r_1}$ and $\vec{r_2}$, and define the environment variables of a charge trap and its near-by electron reservoir as $\vec{r_3}$ and $\vec{r_4}$. We define the Hamiltonian of two qubit system as double well in a two-dimensional layer (z=0) such that $H_0 = \sum_{i=1,2} h_i + C, h_i = \frac{1}{2m}p_i^2 + V(r_i), C = \frac{e^2}{|r_1 - r_2|}, V(x,y) = (\frac{m\omega^2}{2}\frac{1}{4a^2}(x^2 - a^2)^2 + \frac{m\omega^2}{2}y^2)$, where m is the effective mass of an electron in the quantum dot, ω the confinement frequency of confinement potential, and $2a$ the distance between two potential minima. The exchange energy of electron spins between the dots is given by $J_0 = \frac{\hbar\omega}{\sinh(2d^2)}(c(e^{-d^2}I_0(d^2)) + \frac{3}{4}(1 + d^2))$ where $c = \sqrt{\pi/2}(e^2/a_B)/\hbar\omega$, $I_0(x)$ is 0-th order Bessel function, and $d = a/a_B$, $a_B = \sqrt{\hbar/m\omega}$.[25] The interaction Hamiltonian is $V_1 = e^2/\sqrt{|\vec{r_1} - \vec{r_3}|^2} - e^2/\sqrt{|\vec{r_2} - \vec{r_4}|^2} \simeq e^2/r + e^2((\vec{r_1} + \vec{r_2}) \cdot (\vec{r_3} - \vec{r_4}))/r^3$, where $r \simeq \sqrt{|\vec{r_3}|^2} \simeq \sqrt{|\vec{r_4}|^2}$. From the above calculation, the dynamic part of the interaction Hamiltonian between charge trap and qubit system is given by $V_1(\vec{r_1}, \vec{r_2}) = e^2 \frac{(x_1+x_2)(x_3-x_4)+(y_1+y_2)(y_3-y_4)}{r^3}$, where we set z_1 and z_2 to zero. Using the Heitler-London approximation, we define, $|S\rangle$ and $|T\rangle$ to be singlet and triplet states such that, $|S\rangle = (|12\rangle + |21\rangle)/\sqrt{2(1 + S^2)}, |T\rangle = (|12\rangle - |21\rangle)/\sqrt{2(1 - S^2)}$, where, $S^2 = |\langle 1|2\rangle|^2 = \exp(-2d^2)$ is an overlap integral and
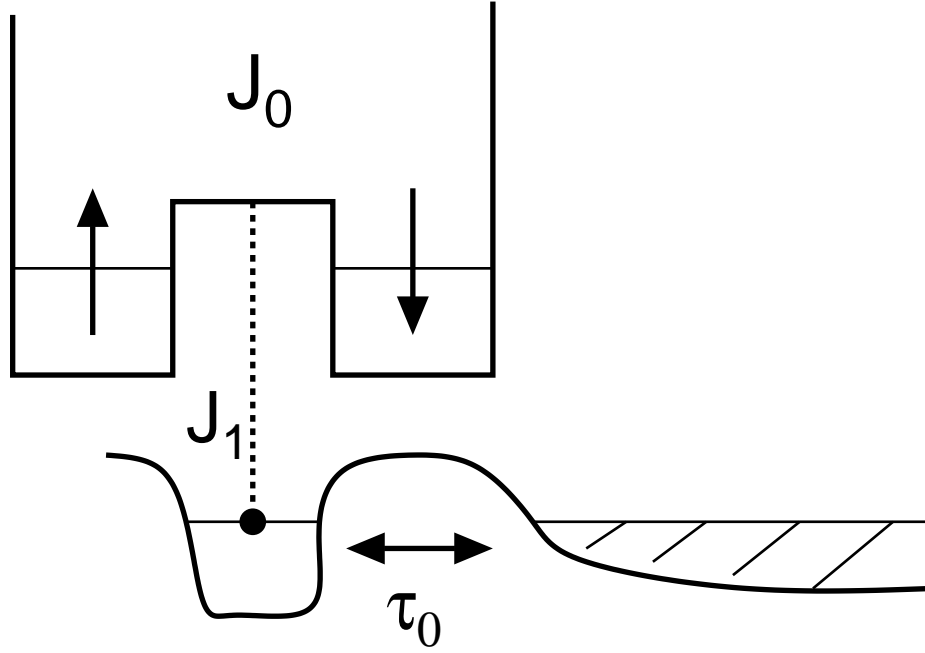
Fig. 1. Scheme of coupled qubit system and charge trap

$|1\rangle = \sqrt{\frac{m\omega}{\pi\hbar}}e^{-m\omega((x-a)^2+y^2)/2\hbar}, |2\rangle = \sqrt{\frac{m\omega}{\pi\hbar}}e^{-m\omega((x+a)^2+y^2)/2\hbar}$. When the trap's dipole vector aligns in y-direction, the fluctuation of the exchange coupling does not exist. Then, the amount of fluctuation due to a single charge trap is written by, $J_{1\text{single}} = \frac{\hbar\omega}{\sinh(2d^2)}\frac{3}{2d^2}(\frac{e^2 a(x_3-x_4)}{\hbar\omega r^3})^2$.[25] For many traps, the magnitude of fluctuation in the exchange coupling becomes as follows. The dipole vector of BCFs is $p(\sin\theta_i\cos\phi_i, \sin\theta_i\sin\phi_i, \cos\theta_i)$ at distance r_i, where p is distance between a trap and electron reservoir. Then the magnitude of fluctuation is given by $J_1 = \sum J_{1\text{single}} = \sum_i \frac{1}{\hbar\omega}\frac{3}{2}\frac{1}{d^2}(\frac{e^2 p\sin\theta_i\cos\phi_i a}{r_i^3})^2\frac{1}{\sinh(2d^2)} = \frac{2\pi}{3}\frac{e^4}{\hbar\omega}\frac{1}{\sinh(2d^2)}\frac{N_i a^2 p^2}{r_d^3 d^2}$, where N_i is the density of charge traps and r_d is the distance where the dipole approximation becomes invalid $(a < r_d)$.

Next, we examine the time evolution of qubit density matrix. The time evolution of the qubit system is defined by the unitary operator

$$U(t) = e^{-\frac{i}{\hbar}\int_0^t J(\tau)\mathbf{S_1}\cdot\mathbf{S_2}d\tau}. \tag{1}$$

The exchange coupling operator is expressed by using permutation operator P_{12} such that

$$\mathbf{S_1}\cdot\mathbf{S_2} = \frac{P_{12}+1}{2}. \tag{2}$$

P_{12} has following properties:

$$P_{12}^{2m} = 1, P_{12}^{2m+1} = P_{12}, \tag{3}$$

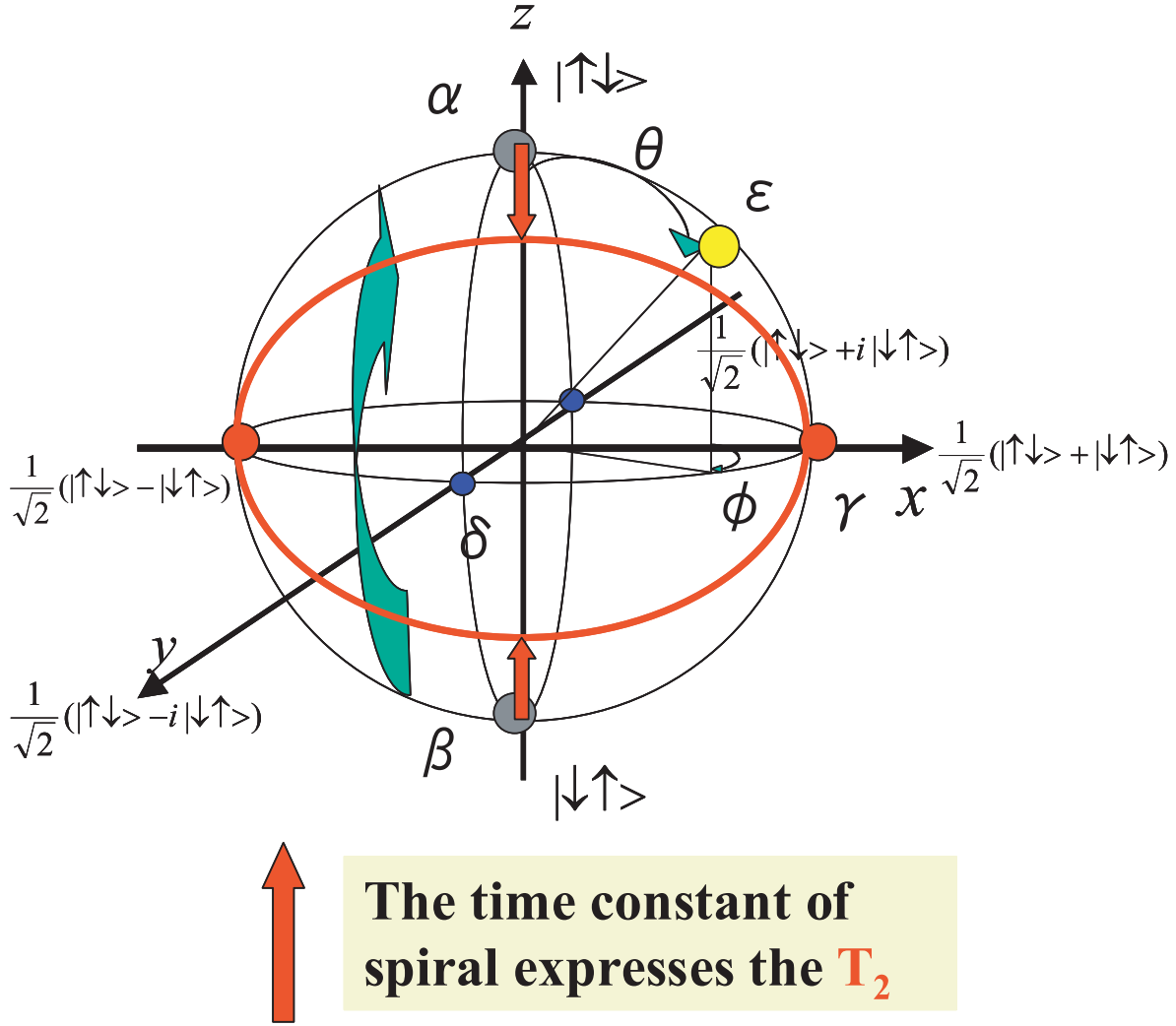where m is integer. We assume the fluctuation in the BCF obeys the random telegraph-type

Fig. 2. Scheme of Bloch sphere with pointing moving quantum state. (θ,ϕ),$\alpha=(0,0)$,$\beta=(0,\pi)$,$\gamma=(\pi/2,0)$,$\delta=(\pi/2,\pi)$,$\epsilon=(\pi/2,\pi/2)$.

noise. $J(t) = J_0 + J_1 X(t)$, where $X(t)$ takes the value of 1 or 0 with characteristic time τ. The density matrix at time t is given by $\rho(t) = \langle U(t)\rho(t=0)U^{\dagger}(t)\rangle$, where $U(t)$ is unitary operator which describes the time evolution in terms of the bases $|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle$ and $\langle\rangle$ is the ensemble average about the stochastic process.[20, 26, 27] To accomplish two-qubit gate operation, the restricted subspace spanned by $|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$ is enough because other states are decoherence free from this type of dephasing.In restricted sub-Hilbert space, the initial pure state is given by $|\Psi\rangle = \cos\frac{\theta}{2}|\uparrow\downarrow\rangle + \sin\frac{\theta}{2}e^{-i\phi}|\downarrow\uparrow\rangle$, and the quantum state is represented by using a Bloch sphere as shown in Fig. 2. We obtain the density matrix at time t after taking the ensemble average and calculate T_2 as had been done for a single qubit system.[20, 26, 27] The states on one-dimensional line that connects two maximally entangled states $(\frac{1}{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

and $(\frac{1}{2}(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle))$. do not evolve during the gate operation. The distance from the origin of the Bloch sphere to quantum state represents the purity. The time evolution trace of the quantum state changes from sphere to ellipsoidal because of dephasing. The maximally entangled states $|\Psi\rangle = (|\uparrow\downarrow\rangle \mp |\downarrow\uparrow\rangle)/\sqrt{2}$ do not evolve in time, since they are eigenstates of the Hamiltonian. Fig. 3 (a),(b) shows time dependence of quantum state with some choice of initial condition. We estimate T_2^{-1} for many charge traps with different characteristic time as follows. We assume the transition times between occupied and empty states are equal and the temperature dependence of transition time obeys the thermal activation type $\tau = Ae^{\frac{-W}{k_B T}}$, where W is thermal activation energy and A is characteristic time scale which is independent of temperature. To estimate the effect of many charge traps, we average over the magnitude of fluctuation in the exchange coupling and thermal activation energy. Since the dephasing rate by a single trap is $J_1^2 \tau \hbar$ for weak coupling and $\hbar/2\tau$ for strong coupling,[19, 20] total T_2^{-1} is given by $T_2^{-1} = \frac{3k_B T}{2W_0} J_1$ where $1/W_0$ is distribution of thermal activation energy. We define the gate operation time as $\tau_p = \pi/J_0$. Then, the gate quality factor is given by $Q = T_2/\tau_p \simeq \frac{2W_0}{3\pi k_B T}\frac{J_0}{J_1}$.

Next, we examine the quantum information quantities of the coupled qubit system. First, we examine the criterion of entanglement. This criterion comes from the negativity of minimum eigenvalue of a partially transposed qubit density matrix.[28, 29] Fig. 4 shows time dependence of criterion of entanglement. For Fig.4-6 we set $T_2/\tau = 2$. Fig. 5 shows time dependence of fidelity. The minimum eigenvalue oscillates with time. When $\sin\theta\cos\phi = 0$, separable states, $|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$, appear between entangled states during time evolution. Whereas, the qubits are always entangled if $\sin\theta\cos\phi \neq 0$. Next, we study the fidelity of $\rho(t)$ relative to $\rho(t = 0)$, defined by $\mathcal{F}(t) \equiv Tr[\rho(t)\rho(0)]$, which shows accuracy of the quantum gate.[30, 31] Starting from the maximally entangled state, fidelity is 1. For more general initial conditions, fidelity is given by

$$
\begin{aligned}
\mathcal{F} &= \frac{1}{2} + \frac{1}{2}\cos(\frac{1}{\hbar}J_0 t)e^{-t/T_2}\cos^2\theta \\
&+ \frac{1}{2}\cos(\frac{1}{\hbar}J_0 t)e^{-t/T_2}\sin^2\theta\sin^2\phi \\
&+ \frac{1}{2}\sin^2\theta\cos^2\phi.
\end{aligned}
\tag{4}
$$

Finally, we examine the purity $(\mathcal{P} = Tr(\rho(t)^2))$, which is related to linear entropy of qubit system as $S_{lin} = 1 - \mathcal{P}$.[30, 32] The analytical expression of purity is

$$
\begin{aligned}
\mathcal{P} &= \frac{1}{2} + \frac{1}{2}\sin^2\theta\cos^2\phi \\
&+ \frac{1}{2}(\sin^2\theta\sin^2\phi + \cos^2\theta)e^{-2t/T_2}.
\end{aligned}
\tag{5}
$$

Figure 6 shows the time dependence of purity. The purity decreases monotonically, which means that the entropy of the qubit increases due to the dephasing. The initially pure qubit state becomes a mixed state, namely, $\mathcal{P}(t = 0) = 1$ becomes $\mathcal{P}(t = \infty) = \frac{1}{2} + \frac{1}{2}\sin^2\theta\cos^2\phi$.
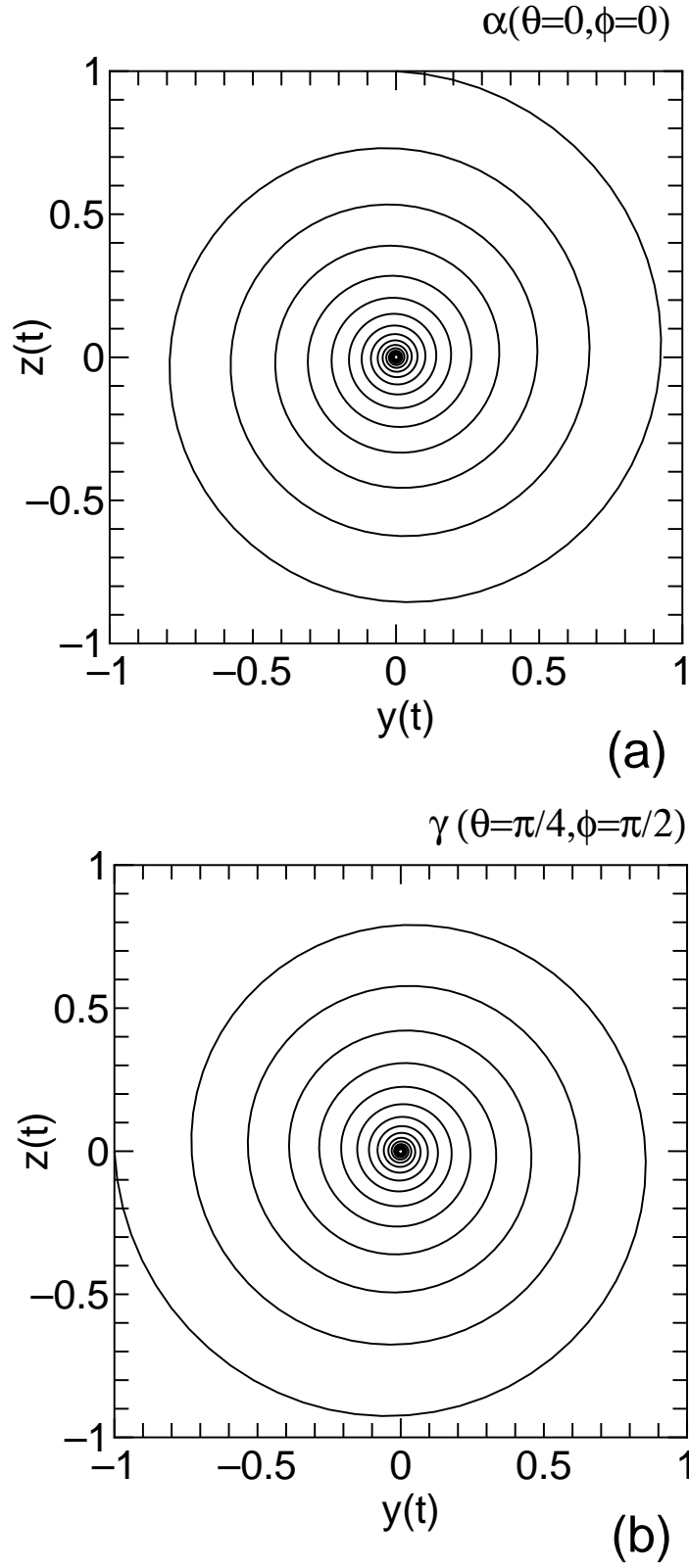
$\alpha(\theta=0,\phi=0)$

(a)

$\gamma\,(\theta=\pi/4,\phi=\pi/2)$

(b)

Fig. 3.   Time depnedence of quantum state. (a) initial
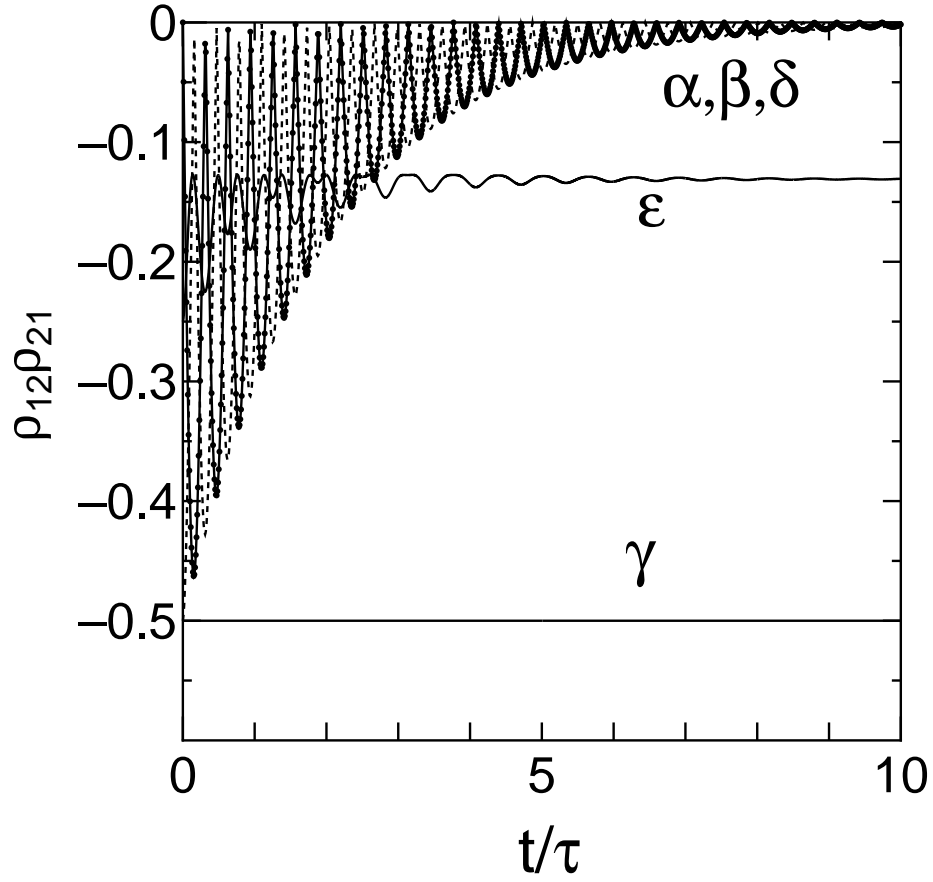state $\alpha=(\theta=0,\phi=0)$. (b) initial state $\beta=(\theta=\theta/4,\phi=\pi/2)$.
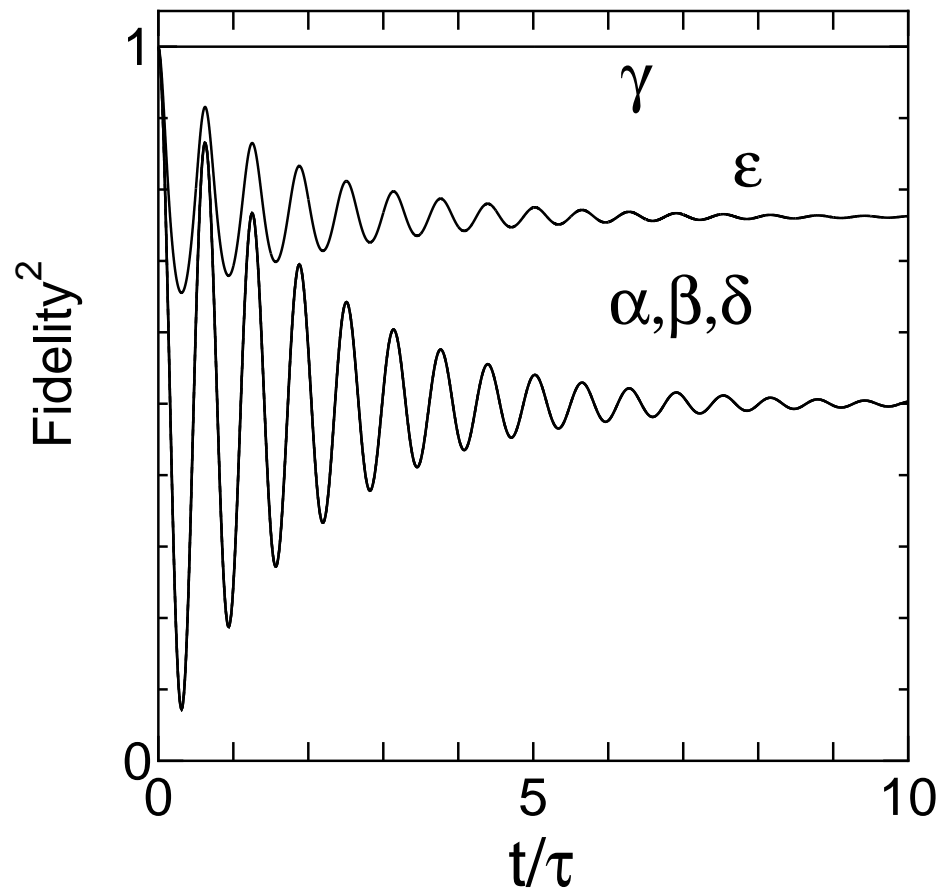
Fig. 4.   Criterion of entanglement.

Fig. 5. Time dependence of fidelity.

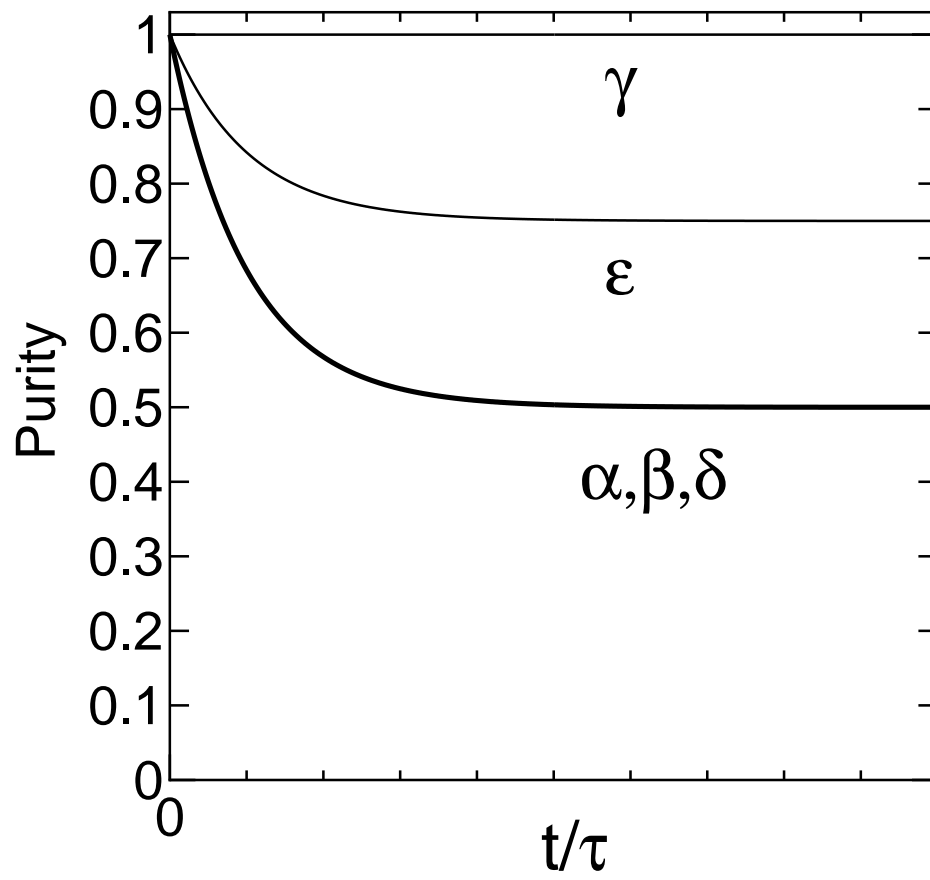Fig. 6.   Time dependence of purity.

References

1) Y. Nakamura, Yu. A. Pashkin and J. S. Tsai : Nature **398** (1999) 786.

2) T. Hayashi, T. Fujisawa, H-D. Cheong, Y-H. Jeong and Y. Hirayama : Phys. Rev. Lett. **91** (2003) 226804.

3) T. Tanamoto : Phys. Rev. A **61** (2000) 22305.

4) D. Loss and D. P. DiVincenzo : Phys. Rev. A **57** (1998) 120.

5) Y. Makhlin, G. Schön and Shniriman : Nature **398** (1999) 305.

6) Y. Makhlin, G. Schön and A. Shnirman : Rev. Mod. Phys. **73** (2001) 357.

7) D. A. Averin : Solid State Commun. **105** (1998) 659.

8) J. E. Mooij, T. P. Orland, L. Levitov, L. Tian, C. H. van der Wal and S. Lloyd : Science **285** (1999) 1036.

9) G. Falci, R. Fazio, G. M. Palma, J. Siewert and V. Verdral : Nature **407** (2000) 355.

10) V. Bouchiat, D. Vion, P. Joyez, D. Esteve and M. H. Devoret : Phys. Scr. **T76** (1998) 165.

11) J. R. Friedman, V. Patel, W. Chen, S. K. Tolpyo and J. E. Lukons : Nature **406** (2000) 43.

12) A. Aassime, G. Johansson, G. Wendin, R. J. Schoellcopf and P. Delsing : Phys. Rev. Lett. **86** (2000) 3376.

13) P. L. Lafarge, P. Joyez, H. Pothier, A. Cleland, T. Holst, D. Esteve, C. Urbina and M. H. Devoret : C. R. Acad. Sci. Paris, **314** (1992) 883.

14) A. B. Zorin, F.-J. Ahlers, J. Niemeyer, T. Weimann, H. Wolf, V. A. Krupenin and S.V. Lotkhov : Phys. Rev. B **53** (1996) 13682.

15) G. Zimmerli, T. M. Eiles, R. L. Kautz and J. M. Martinis : Appl. Phys. Lett. **61** (1992) 13.

16) C. Kurdak, C. J. Chen, D. C. Tsui, S. Parihar, S. Lyon and G. W. Weimann : Phys. Rev. B **56** (1997) 9813.

17) T. Fujisawa and Y. Hirayama : Appl. Phys. Lett **77** (2000) 543.

18) P. Dutta and P. H. Horn : Rev. Mod. Phys. **53** (1981) 497.

19) T. Itakura and Y. Tokura : Phys. Rev. B **67** (2003) 195320.

20) T. Itakura and Y. Tokura : J. Phys. Soc. Jpn. **72** (2003) 2726.

21) T. Fujisawa, D, G. Austing, Y. Tokura, Y. Hirayama and S. Tarucha : Nature **419** (2002) 278.

22) T. Fujisawa, Y. Tokura and Y. Hirayama : Phys. Rev. B **63** (2002) 081304.

23) R. de Sousa, X. Hu, S. Das Sarma : Phys. Rev. A **64** (2001) 042307.

24) G. Burkard and D. Loss : Phys. Rev. Lett. **88** (2002) 047903.

25) G. Burkard, D. Loss and D. P. DiVincenzo : Phys. Rev. B **59** (1999) 2070.

26) R. Kubo : Scottish Universities Summer School, D. ter Haar, ed., Plenum Press, New York, (1961).

27) A. Papoulis : Proceedings of Transactions of 9th Prague Conference on Information Theory, (1982).

28) A. Peres : Phys. Rev. Lett. **77** (1996) 1413.

29) M. Horodecki, P. Horodecki and R. Horodecki : Phys. Lett. A **223** (1996) 1.

30) J. F. Poyatos, J.I. Cirac and P. Zoller : Phys. Rev. Lett. **78** (1997) 390.

31) M. A. Nielsen and I. L. Chuang : "Quantum Computation and Quantum Information" Cambridge University Press, (2000)

32) T. Miyadera and A. Shimizu : quant-ph/0109065